

PHASE-COMPENSATED CUBE CORNER IN LASER INTERFEROMETRY

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DESCRIPTION OF RELATED ART

[0001] The conventional cube corner (CC) retroreflector has the property that any ray entering the effective aperture will be reflected and emerge from the entrance/exit face parallel to itself, but with opposite direction of propagation. This property is, within acceptance angle limits, independent of the orientation of the retroreflector. Retroreflectors therefore find frequent application in situations where angular orientation is difficult or impossible to control and where a mirror would therefore be unsatisfactory. Figs. 1A and 1B illustrate a side and a front view of a conventional CC retroreflector 10.

[0002] Some conventional CC retroreflectors use the principle of total internal reflection (TIR) for its operation. The use of TIR limits the acceptable incident angles for the CC retroreflector. The use of TIR also changes the phase difference between the S and P components so the light exiting the CC retroreflector has a different polarization state, both in orientation and ellipticity, than the light entering the CC retroreflector. Experiments have shown that in solid BK-7 CC retroreflectors using TIR (without any coating), incident linearly polarized light will exhibit about 8.4 degrees of relative rotation of azimuth angle and about 11 degrees of ellipticity. Azimuth is defined as the angle between the major axis of the polarization ellipse to horizontal or vertical reference axes, and tangent of ellipticity is defined as the ratio of minor to major axes of the polarization ellipse.

[0003] The limitation on the incident angle can be removed, at the cost of a reduction in reflective efficiency, by applying a reflective coating to the rear reflecting surfaces (e.g., faces 1, 2, and 3). Thus, some conventional CC retroreflectors have their rear reflecting surfaces coated with silver and then with paint to prevent oxidation of the silver. Nonetheless, the use of the reflecting coating still causes the light exiting the CC retroreflector to have a different polarization state than the light entering the CC retroreflector. Experiments have shown that in solid BK-7 CC retroreflectors with silver coated rear reflecting surfaces, incident linearly polarized light will exhibit about 6.0 degrees of relative rotation of azimuth angle and about 1.0 degree of ellipticity. The paint used to

coat the silver is also known to outgas in certain environments and contaminate the surrounding components.

[0004] Fig. 2 illustrates a conventional plane mirror interferometer system 20. A laser head 22 generates a coherent, collimated light beam consisting of two orthogonally polarized frequency components. One frequency component f_A (e.g., a measurement beam having a P-polarization) enters the interferometer's measurement path while the other frequency component f_B (e.g., a reference beam having an S-polarization) enters the interferometer's reference path.

[0005] In the measurement path, a polarizing beam-splitter 24 transmits frequency component f_A to a measurement plane mirror 26 mounted to a moving stage. Since frequency component f_A passes through a quarter-wave plate 28, the returning polarization is rotated 90 degrees and the newly S-polarized frequency component f_A is reflected by polarizing beam-splitter 24 to CC retroreflector 10. Retroreflector 10 directs frequency component f_A again to polarizing beam-splitter 24, which again reflects frequency component f_A to measurement mirror 26. Again, since frequency component f_A passes through quarter-wave plate 28, the returning polarization is rotated 90 degrees and the newly P-polarized frequency component f_A is transmitted through polarizing beam-splitter 24 onto a receiver 34.

[0006] In the reference path, polarizing beam-splitter 24 reflects frequency component f_B to a reference plane mirror 30. Since frequency component f_B passes through a quarter-wave plate 32, the returning polarization is rotated 90 degrees and the newly P-polarized frequency component f_B is transmitted through polarizing beam-splitter 24 to CC retroreflector 10. CC retroreflector 10 directs frequency component f_B through beam-splitter 24 to reference mirror 30. Again, since frequency component f_B passes through quarter-wave plate 32, the returning polarization is rotated 90 degrees and the newly S-polarized frequency component f_B is reflected by polarizing beam-splitter 24 coaxially with frequency component f_A onto receiver 34.

[0007] The importance of maintaining the polarization states through CC retroreflector 10 can be seen in Fig. 2. If CC retroreflector 10 changes the orientations of frequency components f_A and f_B , then parts of frequency components f_A and f_B would leak through polarizing beam-splitter 24, which can cause a decrease in the accuracy of the distance measurements.

[0008] A paper by Paul Mauer discloses that a three-layer phase-compensating film stack can be formed on the rear reflecting surfaces of a CC retroreflector to compensate the phase shift due to TIR. “Phase Compensation of Internal Reflection” by Paul Mauer, J. Opt. Soc. Am. 56, 1219 (1966). However, the paper states that although the ellipticity of the light is preserved, the orientation of the light is changed. “Although a coating of this type compensates phase shift on total internal reflection, there remains, in general, a rotation of the plane of polarization when light is returned from a phase-compensated corner cube.” *Id.* at 1221. Reading this paper, one skilled in the art would be discouraged from using the phase-compensated cube corner in an interferometer system because the paper states that the polarization orientation is changed by the phase-compensated cube corner. As described above, change in the polarization orientation is known to decrease the accuracy of the interferometry system.

[0009] In a series of lecture notes for a UCLA Extension class, Philip Baumeister cites the Mauer paper for describing the use of a three-layer film stack to reduce the differential phase shift upon reflection. “Optical Coating Technologies” by Philip Baumeister, UCLA Extension (1998). In Fig. 1-122 of these lecture notes, Baumeister shows a differential phase shift of approximately 0 degree for a normal angle of incidence when the three-layer film stack is applied to the three rear reflecting surfaces of the cube corner. Reading this paper, one skilled in the art would also read the Mauer paper described above. Thus, for the same reasons described above, one skilled in the art would be discouraged from using the phase-compensated cube corner in an interferometer system because the Mauer paper states that the polarization orientation is changed by the phase-compensated cube corner.

[0010] Thus, what is needed is a CC retroreflector that addresses the relative polarization rotation, ellipticity, reflective efficiency, and contamination problems of a conventional CC retroreflector.

SUMMARY

[0011] In one embodiment of the invention, a phase-compensating cube corner retroreflector includes three rear reflecting surfaces. The first and the third rear reflecting surfaces can be coated with a phase-compensating film stack that induces $2n\pi$ phase difference upon reflection, where n is an integer including 0. Alternatively, all three rear reflecting surfaces can be coated with a phase-compensating film stack that induces $n\pi$ phase difference when

handling linearly polarized light, or $2n\pi$ phase difference when handling linearly or circularly polarized light. So coated, the phase-compensating cube corner preserves the polarization orientation and ellipticity of the incident light.

[0012] In one embodiment, an interferometer system includes a phase-compensating cube corner. The phase-compensating cube corner directs light to and from other optical elements, including polarizing beam-splitters, mirrors, and quarter-wave plates.

BRIEF DESCRIPTION OF THE DRAWINGS

[0013] Figs. 1A and 1B illustrate a side and a front view of a conventional cube corner retroreflector.

[0014] Fig. 2 illustrates a conventional plane mirror interferometer system.

[0015] Figs. 3 to 8 illustrate the derivation of the polarization orientation and ellipticity with an uncoated conventional cube corner in one embodiment of the invention.

[0016] Figs. 9 to 12 illustrate the derivation of the polarization orientation and ellipticity with a phase-compensated cube corner having 0 degree phase difference in one embodiment of the invention.

[0017] Figs. 13 to 16A illustrate the derivation of the polarization orientation and ellipticity with a phase-compensated cube corner having 180 degree phase difference in one embodiment of the invention.

[0018] Figs. 16B and 16C illustrate the reflected phase from phase-compensated cube corners having 0 degree phase difference in embodiments of the invention.

[0019] Fig. 17A illustrates a plane mirror interferometer system in one embodiment of the invention.

[0020] Figs. 17B and 17C illustrate interferometer systems in embodiments of the invention.

[0021] Figs. 18A, 18B, and 18C illustrate a differential interferometer system in one embodiment of the invention.

[0022] Fig. 19 illustrates a single beam interferometer system in one embodiment of the invention.

[0023] Figs. 20A, 20B, and 20C illustrate a high-resolution interferometer system in one embodiment of the invention.

[0024] Fig. 21 illustrates a linear interferometer system in one embodiment of the invention.

DETAILED DESCRIPTION

[0025] In one embodiment of the invention, a dielectric multilayer thin film coating is deposited on all reflecting surfaces or only the first and the third reflecting surfaces of a solid BK-7 cube corner. The thin film stack is designed specifically to preserve upon reflection the azimuth and ellipticity angles of the incident polarization. The thin film coating reduces polarization rotation from about 6 degrees (for silver coated cube corner) to about 0.5 degree, and ellipticity from about 1.0 degree to about 0.5 degree for a linearly polarization state. Additionally, since all reflections are accomplished with total internal reflection (TIR), the net irradiance efficiency is near 100%. Furthermore, problems associated with outgassing are alleviated because the rear reflecting surfaces of the cube corner do not need to be painted as the dielectric materials are inert.

Introduction

[0026] Reflection processes in general induce change to the polarization state of light. Polarization states change on reflection as electric field components undergo relative phase retardation and/or coordinate transformation. The extent of the change depends on the incident angle and the polarization as well as on the optical properties of the surface.

[0027] To be described herein, the polarization effects associated with total internal reflections (TIR) in solid cube corners are considered. Using Stokes parameter representation, a numerical model is created to make predictions and develop ways to preserve ellipticity and azimuth angle of the initial polarization.

Uncoated TIR Cube Corner

[0028] For an uncoated cube corner, each reflection is TIR with a phase retardation Δ of 45.2 degrees between the S and the P polarization states. For an incident S-polarized beam propagating in negative z direction into face 1 with a face folded edge perpendicular to the horizontal plane as shown in Fig. 3, the beam undergoes three reflections and finally emerges from face 3 propagating in the positive z direction. For the first reflection on face 1, the unit

normal vector for the plane of incidence is given by:

$$N_1 = 0.500i + 0.866j + 0.000k, \quad (0.1)$$

where i , j and k are unit vectors in the laboratory Cartesian frame of Fig. 3. In a Cartesian coordinate frame defined by N_1 , the S-polarization makes an angle of -30 degrees with the y -axis of this frame as shown in Fig. 4. The azimuth angle is negative since it is measured counterclockwise from the y -axis. In this rotated frame, the y -axis is perpendicular while the x - and z -axes lie in the plane of incidence of face 1. The incident unit magnitude polarization vector in this frame is represented by its components a_1 and a_2 . The values for components a_1 and a_2 are:

$$a_1 = -0.500 \quad (1)$$

$$a_2 = 0.866. \quad (2)$$

[0029] With these values of components a_1 , a_2 and the azimuth $\psi (= \psi_0) = -30$ degrees in face 1 coordinate frame and looking into the beam as shown in Fig. 4, it is possible to use equation II.12 of Appendix II to calculate the phase difference δ between components a_1 and a_2 of the input beam as follows:

$$(a_1^2 - a_2^2)\tan 2\psi = 2a_1a_2\cos\delta. \quad (II.12)$$

[0030] Henceforth, all references to coordinate frames are from the view of looking into a beam. It is important that in equation II.12 the azimuth angle ψ is measured from the y -axis. This is only true when $|a_2| > |a_1|$. Solving for the phase difference δ , it is determined that:

$$\delta = \delta_0 = 180 \text{ deg.} \quad (2.1)$$

[0031] This value for phase difference δ_0 is then added to the relative phase retardation Δ , which is 45.2 degrees for uncoated TIR reflection at 54.7 degrees of incidence, to enable the calculation of ellipticity χ_1 and azimuth angle ψ_1 of the reflected beam from face 1 as follows:

$$\delta_1 = \Delta + \delta_0 = 225.2 \text{ deg.} \quad (2.2)$$

$$\tan\alpha_1 = a_2/a_1 = -1.732. \quad (2.3)$$

Phase difference δ_1 is post reflection relative phase retardation between components a_1 and a_2 . Additionally due to the reflection being TIR, $\tan\alpha_1 = \tan\alpha_0$. The calculation for azimuth angle ψ_1 proceeds with the reapplication of equation II.12 using the above value for phase difference δ_1 as follows:

$$\tan 2\psi_1 = 2\tan\alpha_1 \cos\delta_1 / (1 - \tan^2\alpha_1) \quad (3)$$

$$\psi_1 = -25.3 \text{ deg.}$$

[0032] Next using the Stokes parameter S_3 of equation III.15 in Appendix III, the ellipticity χ_1 is calculated as follows:

$$S_3 = \sin 2\chi = \{2\tan\alpha/(1 + \tan^2\alpha)\} \sin\delta \quad (4)$$

$$\chi_1 = \sin^{-1}[\{2\tan\alpha_1/(1 + \tan^2\alpha_1)\} \sin\delta_1]/2 \quad (5)$$

$$\chi_1 = 18.96 \text{ deg.}$$

In equation 5, the tangent of ellipticity angle χ_1 is the ratio of the minor to the major axis of the elliptical polarization. See equation III.0.b in Appendix III. Fig. 5 illustrates the reflected polarization state relative to the plane of incidence of face 1.

[0033] As was the case for the incident beam, the azimuth angle ψ_1 is negative with respect to the plane of incidence for face 1. The next step is to calculate azimuth angle ψ_1 relative to the plane of incidence of face 2 (ellipticity angle χ_1 remaining unchanged with the rotation of coordinate frame). This is necessary since the reflected polarization from face 2 must be made relative to the plane of that face. To perform the coordinate transformation from face 1 to face 2, it is necessary to first know the unit normal vector for the plane of incidence of face 2. The unit normal vector for the plane of incidence of face 2 is:

$$N_2 = 0.000i + 0.577j + 0.816k. \quad (5.1)$$

Then taking the dot product of normal vectors N_1 and N_2 , the angle ϕ_{12} between planes for faces 1 and 2 is given by:

$$\phi_{12} = 60 \text{ deg.} \quad (5.2)$$

[0034] Fig. 6 illustrates the relative orientation of the two frames of reference where y_1, x_1

and y_2, x_2 are coordinate frames for faces 1 and 2 respectively. It is evident from Fig. 6 that the azimuth angle ψ_{12} of the elliptically polarized state with respect to y_2 axis is given by,

$$\psi_{12} = \psi_1 + 60 = 60 - 25.3 = 34.7 \text{ deg.} \quad (5.3)$$

The subscript 12 refers to the azimuth angle relative to frame 2 for the reflected beam from face 1. As shown in Fig. 6, the azimuth angle ψ_{12} is a positive quantity (whereas azimuth angle ψ_1 was negative). Once azimuth angle ψ_{12} and the ellipticity angle χ_1 are determined for y_2, x_2 frame, the associated components a_1 and a_2 are also calculated. As described earlier with the input polarization for the first reflection, new values for components a_1, a_2 and phase difference δ_{12} are needed in advance of the second reflection. These quantities are determined from the Stokes parameter S_1 of equation III.13 from Appendix III and equation II.12 from Appendix II as follows:

$$S_1 = (1 - \tan^2 \alpha) / (1 + \tan^2 \alpha). \quad (6a)$$

Alternatively, these quantities are determined from the normalized form of equation III.10 from Appendix III as follows:

$$S_1 = \cos 2\chi \cos 2\psi. \quad (6b)$$

In equation 6b, the azimuth angle ψ is always measured from the x-axis. Therefore, its value equal to:

$$\psi = 90 - \psi_{12}. \quad (6.1)$$

Solving equation 6a for $\tan \alpha$ and combining with equation 6b provide:

$$\begin{aligned} \tan \alpha_{12} &= \{(1 - \cos 2\chi_1 \cos 2\psi) / (1 + \cos 2\chi_1 \cos 2\psi)\}^{1/2} \\ &= 1.331. \end{aligned} \quad (7)$$

From equation II.12:

$$\cos \delta_{12} = \{(1 - \tan^2 \alpha_{12}) / \tan \alpha_{12}\} \tan 2\psi_{12} / 2 \quad (8)$$

$$\delta_{12} = 140.2 \text{ deg.}$$

[0035] As a result of measuring the azimuth from frame 2 (given by azimuth angle ψ_{12}), new

values are assigned to $\tan\alpha$ and δ (see equations 7 and 8). So $\tan\alpha$ and δ are inherently dependent on the reference frame in which the azimuth angle ψ is measured. For reflection 2 from uncoated face 2, the relative phase retardation Δ on $\tan\alpha_{12}$ (or equivalently on components a_1 and a_2) is once again 45.2 degrees. Then it follows that the phase difference δ_2 is equal to:

$$\delta_2 = \Delta + \delta_{12} = 275 \text{ deg.} \quad (8.1)$$

[0036] Using calculation methods developed earlier for reflection 1, and applying equations 3 and 5 respectively for the azimuth angle ψ_2 and the ellipticity angle χ_2 , it is determined that:

$$\tan 2\psi_2 = 2\tan\alpha_2 \cos\delta_2 / (1 - \tan^2\alpha_2) \quad (8.2)$$

$$\psi_2 = 36.9 \text{ deg.}$$

$$\chi_2 = \sin^{-1}[\{2\tan\alpha_2 / (1 + \tan^2\alpha_2)\} \sin\delta_2] / 2 \quad (8.3)$$

$$= -2.593 \text{ deg.}$$

Not that $\tan\alpha_2 = \tan\alpha_{12}$ for TIR reflection.

[0037] The azimuth angle ψ_{23} with respect to face 3 is next found by noting that the y_2, x_2 and y_3, x_3 frames are relatively rotated by 60 degrees as shown in Fig. 7. This is determined from knowing that the unit normal vector for plane of incidence of face 3 is,

$$N_3 = 0.500i + 0.866j + 0.000k. \quad (8.4)$$

[0038] Hence the dot product of normals N_2 and N_3 yields the angle between x_2 - and x_3 -axes to be 60 degrees. Additionally, x_2 and x_3 axes lie in the plane of incidence for faces 2 and 3 respectively, which is analogous to the fact that previously x_1 and x_2 axes were in the plane of incidences for faces 1 and 2. In this case, however, the azimuth angle ψ_{23} is rotated counterclockwise from y_3 axis and is therefore a negative number calculated as follows:

$$\Psi_{23} = \psi_2 - 60 = -23.1 \text{ deg.} \quad (8.5)$$

[0039] Furthermore, $\tan\alpha_{23}$ is also negative and is calculated using equation 7 as follows:

$$\psi = 90 - \psi_{23} \quad (8.6)$$

$$\tan\alpha_{23} = \{(1 - \cos 2\chi_2 \cos 2\psi)/(1 + \cos 2\chi_2 \cos 2\psi)\}^{1/2} \quad (8.7)$$

$$\tan\alpha_{23} = -2.33$$

[0040] Using equation 8,

$$\cos\delta_{23} = \{(1 - \tan^2\alpha_{23})/\tan\alpha_{23}\}\tan 2\psi_{23}/2 \quad (8.8)$$

$$\delta_{23} = 172.9 \text{ deg.}$$

[0041] For the reflection from face 3, $\tan\alpha_{23}$ and δ_{23} from reflection 2 are utilized as inputs to determine the final ellipticity and azimuth angle. Using calculation methods outlined for reflection 1 and 2, it is determined that:

$$\delta_3 = \Delta + \delta_{23} \quad (8.9)$$

$$= 218.1 \text{ deg.,}$$

where $\Delta = 45.2$ degrees and $\tan\alpha_3 = \tan\alpha_{23}$. Then from equations 3 and 5,

$$\tan 2\psi_3 = 2\tan\alpha_3 \cos\delta_3 / (1 - \tan^2\alpha_3) \quad (8.10)$$

$$\psi_3 = -19.8 \text{ deg.}$$

$$\chi_3 = \sin^{-1}[\{2\tan\alpha_3/(1 + \tan 2\alpha_3)\}\sin\delta_3]/2 \quad (8.11)$$

$$\chi_3 = 13.3 \text{ deg.}$$

[0042] Then converting the azimuth angle to laboratory frame as shown in Fig. 8,

$$\psi_{\text{lab}} = 30 + \psi_3. \quad (8.12)$$

$$= 10.2 \text{ deg.}$$

[0043] So after three TIR reflections, the initial polarization has transformed from its linear vertical state to one that has rotated 10.2 degrees and with ellipticity of 13.3 degrees. These computed values are in close agreement with experimental results:

	Predicted (deg.)	Measured (deg.)
Azimuth	10.2	8.4
Ellipticity	13.3	11.0

Table 1 - Predicted and measured azimuth and ellipticity angles

[0044] Similar calculation can also be made for P-polarized light.

Phase Compensated TIR Cube Corner

[0045] Reflection from a TIR cube corner with uncoated faces results in a transformation of the input polarization state from linear to an elliptical state. This is a consequence of non-zero phase retardation that occurs on an uncoated surface at the TIR angle of 54.7 degrees (the incident angle for each reflection in a cube corner). The idea of phase compensated coating is to introduce an interference coating stack on the TIR surface with certain phase thicknesses and refractive indices such as to reduce the reflected phase retardation from 45.2 to 0 or 180 degrees.

[0046] In order to produce optical interference within the coating stack, refractive indices for coating materials must be chosen in such a way that the internal transmitted angles be less than their respective critical angles. This way, the phase thicknesses for all layers in the stack will influence the net change in the reflected phase. With the aid of a thin film analysis program, various coating stacks are designed and investigated below.

Case I: Coating phase difference is zero degree (i.e., $\Delta = 180$ degrees)

[0047] In one embodiment, the thin film stack on the cube corner induces 0 degree phase difference for each of the three reflections that occur on a cube corner. The term phase difference follows the convention used in thin film calculation whereas phase retardation Δ follows the convention used in ellipsometry. The design is shown in Table 2:

Layer#	Material	Index (at 633 nm)	QWOT (nm)
1	SiO ₂	1.46	815
2	TiO ₂	2.45	1066
3	SiO ₂	1.46	1090
4	TiO ₂	2.45	1702

Table 2 - Zero degree phase difference

QWOT is an optical thickness equal to $4 \cdot N \cdot T$, where N is the refractive index and T is the physical thickness. Please note that the higher TiO₂ index is achieved with ion-assist deposition process. Here layer 1 is the first layer on the BK-7 surface. Fig. 9 illustrates the phase change with the coating of Table 2.

[0048] This coating may now be incorporated in the polarization model developed for the analysis of the uncoated TIR cube corner to calculate the polarization state produced from a phase-compensated cube corner. Before continuing, however, it is important to resolve sign convention differences between thin film and polarization analysis. In the thin film convention, coatings that produce zero degree phase difference between the S and the P polarization states do not alter the sense of polarization. For example, incident right circular will remain right circular after reflection. However, the same phenomenon is described in polarization analysis as having flipped in the sense of polarization rotation from right to left circular. This happens because in polarization analysis the coordinate frame is always defined looking into the beam. Therefore, incident right circular polarization (looking into the beam) appears as left circular after reflection when viewing into the beam, even with zero degree reflected phase change. It thus appears, from the point of view of polarization analysis, that the coating induces exactly 180 degrees phase change on reflection between S and P and not zero degree as was predicted by the thin film convention. Even at normal incidence, polarization analysis predicts a change in the handedness of the polarization on reflection from right to left or visa versa, when physically there is no phase difference between S and P at normal incidence.

[0049] With this understanding, the TIR cube corner with the phase compensating coating of Table 2 is hereafter investigated.

Input polarization

[0050] As in the earlier example of uncoated cube corner, the input state is S polarized with respect to laboratory frame. Thus, components a_1 , a_2 and phase difference δ_0 have the following values:

$$a_1 = -0.500 \quad (8.13)$$

$$a_2 = 0.866 \quad (8.14)$$

$$\delta_0 = 180. \quad (8.15)$$

Reflection 1

[0051] For reflection from face 1:

$$\Delta = 180 \text{ deg.} \quad (8.16)$$

$$\tan\alpha = -0.866/0.500 = -1.732. \quad (8.17)$$

$$\cos(\Delta + \delta_0) = \{(1 - \tan^2\alpha)/\tan\alpha\} \tan 2\psi_1/2 \quad (9)$$

$$\sin 2\chi_1 = \{2\tan\alpha/(1 + \tan^2\alpha)\} \sin(\Delta + \delta_0) \quad (10)$$

Solving (9) for ψ_1 and (10) for χ_1 ,

$$\psi_1 = 30 \text{ deg.} \quad (10.1)$$

$$\chi_1 = 0 \text{ deg.} \quad (10.2)$$

[0052] Fig. 10 shows the reflected polarization state R_1 from face 1. It is noteworthy in this instance that R_1 lies along the x-axis of frame 2. This important result will be utilized later in on embodiment where the phase compensation coating is only applied to two faces of the cube corner.

[0053] With respect to frame 2,

$$\psi_{12} = \psi_1 + 60 = 90 \text{ deg.} \quad (10.3)$$

$$\psi = 90 - \psi_{12} \quad (10.4)$$

$$\tan\alpha_{12} = \{(1 - \cos 2\chi_1 \cos 2\psi)/(1 + \cos 2\chi_1 \cos 2\psi)\}^{1/2} \quad (11)$$

$$= 0$$

$$\cos\delta_{12} = \{(1 - \tan^2\alpha_{12})/\tan\alpha_{12}\}\tan 2\psi'_{12}/2 \quad (12)$$

$$\delta_{12} = 90.$$

[0054] Note that in equation 11, $\tan\alpha_{12} = 0$ and this implies that $|a_2| < |a_1|$. Therefore, the azimuth angle ψ'_{12} in equation 12 is measured with respect to the x -axis or in this case to the x_2 -axis. The value for ψ'_{12} is equal to,

$$\psi'_{12} = 0 \text{ deg.} \quad (12.1)$$

Reflection 2

[0055] For reflection 2, no change is expected on ellipticity and azimuth angle since R_1 lies on the x_2 -axis with no component of the polarization vector existing on the y_2 -axis. This effect only holds true when the incident polarization to the cube corner (i.e., to face 1) is S or P polarized (with respect to laboratory frame). For any other polarization states, a vector component will exist along the y_2 -axis.

$$\Delta = 180 \text{ deg.} \quad (12.2)$$

$$\tan\alpha = \tan\alpha_{12} = 0 \quad (12.3)$$

$$\cos(\Delta + \delta_{12}) = \{(1 - \tan^2\alpha)/\tan\alpha\}\tan 2\psi'_2/2 \quad (13)$$

$$\sin 2\chi_2 = \{2\tan\alpha/(1 + \tan^2\alpha)\}\sin((\Delta + \delta_{12})) \quad (14)$$

Solving equation 13 for the azimuth angle ψ'_2 and equation 14 for the ellipticity angle χ_2 ,

$$\psi'_2 = 0 \text{ deg.} \quad (14.1)$$

$$\chi_2 = 0 \text{ deg.} \quad (14.2)$$

Again note that $\tan\alpha_{12} = 0$. Therefore the azimuth ψ'_2 is measured with respect to x_2 -axis.

[0056] With respect to frame 3, the polarization vector R_2 forms an angle as shown in Fig.

11,

$$\psi_{23} = \psi'_2 + 30 = 30 \text{ deg.} \quad (14.3)$$

[0057] In using the S_1 Stokes parameter in equations 6a and 6b, care must be taken by noting that the azimuth angle in these equations are measured from the x-axis. That is to say,

$$\psi = 90 - \psi_{23} \quad (14.4)$$

$$= 60 \text{ deg.}$$

$$\tan\alpha_{23} = \{(1 - \cos 2\chi_2 \cos 2\psi) / (1 + \cos 2\chi_2 \cos 2\psi)\}^{1/2} \quad (14.5)$$

$$= 1.732$$

$$\cos\delta_{23} = \{(1 - \tan^2\alpha_{23}) / \tan\alpha_{23}\} \tan 2\psi_{23}/2 \quad (14.6)$$

$$\delta_{23} = 180 \text{ deg.}$$

Reflection 3

[0058] For reflection 3,

$$\Delta = 180 \text{ deg.} \quad (14.7)$$

$$\tan\alpha = \tan\alpha_{23} = 1.732. \quad (14.8)$$

$$\cos(\Delta + \delta_{23}) = \{(1 - \tan^2\alpha) / \tan\alpha\} \tan 2\psi_3/2 \quad (15)$$

$$\sin 2\chi_3 = \{2\tan\alpha / (1 + \tan^2\alpha)\} \sin(\Delta + \delta_{23}) \quad (16)$$

Solving equation 15 for the azimuth angle ψ_3 and equation 16 for the ellipticity angle χ_3 ,

$$\psi_3 = -30 \text{ deg.} \quad (16.1)$$

$$\chi_3 = 0 \text{ deg.} \quad (16.2)$$

[0059] The azimuth angle relative to laboratory frame is given by,

$$\psi_{\text{lab}} = 30 + \psi_3 \quad (16.3)$$

$$= 0 \text{ deg.}$$

[0060] With the azimuth angle $\psi_{lab} = 0$ and ellipticity $\chi_3 = 0$, the cube corner with three TIR coatings preserves the state of linear and vertical input polarization.

Case II: Only faces 1 and 3 are coated.

[0061] In one embodiment, only faces 1 and 3 are coated with the coating of Table 2. If the same S-polarization is used as input, then the calculations of the ellipticity χ_1 and azimuth angle ψ_1 up to reflection 2 will be the same as that of Case I. Therefore the Case I results for reflection 1 will be utilized here.

Reflection 2

[0062] As described in reflection 1 of Case I, the reflected polarization R_1 from face 1 lies in the plane of incidence of face 2 as shown in Fig. 10. Thus, regardless of the amount of phase change that occurs from face 2, the reflected state R_2 will always lie in that plane of incidence (i.e., there will be no change in its azimuth angle or ellipticity). Therefore, face 2 could have been left uncoated in Case I and the same end result would have been achieved. This result, however, is only valid for linear S or P input polarization states.

$$\Delta = 45.2 \text{ deg. (for uncoated surface)} \quad (16.4)$$

$$\delta_{12} = 90 \quad (16.5)$$

$$\tan\alpha = \tan\alpha_{12} = 0 \quad (16.6)$$

$$\cos(\Delta + \delta_{12}) = \{(1 - \tan^2\alpha)/\tan\alpha\} \tan 2\psi'_2/2 \quad (17)$$

$$\sin 2\chi_2 = \{2\tan\alpha/(1 + \tan^2\alpha)\} \sin((\Delta + \delta_{12})) \quad (18)$$

Solving equation 17 for the azimuth angle ψ'_2 and equation 18 for the ellipticity angle χ_2 ,

$$\psi'_2 = 0 \text{ deg.} \quad (18.1)$$

$$\chi_2 = 0 \text{ deg.} \quad (18.2)$$

[0063] With respect to frame 3 as shown in Fig. 11, the reflected state R_2 has

$$\psi_{23} = \psi'_2 + 30 = 30 \text{ deg.} \quad (18.3)$$

[0064] Because these values for the azimuth angle ψ'_2 , the ellipticity angle χ_2 , and the azimuth angle ψ_{23} are identical to corresponding reflection 2 values from Case I, same results are also expected for reflection 3 of this case to that of Case I. From the point of view of vertical linear or horizontal linear input states, the two cases (both Case I and II) produce the same results. However, the same results cannot be produced for circularly polarized input states when only the first and the third faces are coated.

Case III: Coating phase difference is 180 degrees (i.e., $\Delta = 0$ degree).

[0065] In one embodiment, the thin film stack on the cube corner induces 180 degrees phase difference on reflection. The coating design is shown in Table 3.

Layer#	Material	Index (at 633 nm)	QWOT (nm)
1	TiO ₂	2.45	1316
2	SiO ₂	1.46	1279
3	TiO ₂	2.45	2595

Table 3 - 180 degree phase difference

Here layer 1 is the first layer on the BK-7 surface. Fig. 13 illustrates the phase change with the coating of Table 3.

[0066] With the 180 degrees phase difference coating on all three faces, the output polarization state from a cube corner is now calculated.

Input Polarization

[0067] The input state is S polarized with respect to laboratory frame.

$$a_1 = -0.500 \quad (18.4)$$

$$a_2 = 0.866 \quad (18.5)$$

$$\delta_0 = 180 \text{ deg.} \quad (18.6)$$

Reflection 1

[0068] For reflection 1,

$$\Delta = 0 \text{ deg.} \quad (18.7)$$

$$\tan\alpha = -0.866/0.500 = -1.732. \quad (18.8)$$

$$\cos(\Delta + \delta_0) = \{(1 - \tan^2\alpha)/\tan\alpha\}\tan 2\psi_1/2 \quad (19)$$

$$\sin 2\chi_1 = \{2\tan\alpha/(1 + \tan^2\alpha)\}\sin(\Delta + \delta_0) \quad (20)$$

Solving equation 19 for the azimuth angle ψ_1 and equation 20 for the ellipticity angle χ_1 ,

$$\psi_1 = -30 \text{ deg.} \quad (20.1)$$

$$\chi_1 = 0 \text{ deg.} \quad (20.2)$$

[0069] Figure 14 shows that the reflected R_1 is collinear with the input state. The apparent magnitude difference between S and R_1 , in Figure 14, is only meant for clarity, since for TIR the reflected amplitude is equal to that of the incident.

[0070] With respect to frame 2,

$$\psi_{12} = \psi_1 + 60 = 30 \text{ deg.} \quad (20.3)$$

$$\psi = 90 - \psi_{12} \quad (20.4)$$

$$\tan\alpha_{12} = \{(1 - \cos 2\chi_1 \cos 2\psi)/(1 + \cos 2\chi_1 \cos 2\psi)\}^{1/2} \quad (20.5)$$

$$= 1.732$$

$$\cos\delta_{12} = \{(1 - \tan^2\alpha_{12})/\tan\alpha_{12}\}\tan 2\psi_{12}/2 \quad (20.6)$$

$$\delta_{12} = 180 \text{ deg.} \quad (20.7)$$

Reflection 2

[0071] For reflection 2,

$$\Delta = 0 \text{ deg.} \quad (20.8)$$

$$\tan\alpha = \tan\alpha_{12} = 1.732 \quad (20.9)$$

$$\cos(\Delta + \delta_{12}) = \{(1 - \tan^2\alpha)/\tan\alpha\}\tan 2\psi_2/2 \quad (21)$$

$$\sin 2\chi_2 = \{2\tan\alpha/(1 + \tan^2\alpha)\}\sin((\Delta + \delta_{12})) \quad (22)$$

Solving equation 21 for the azimuth angle ψ_2 and equation 22 for the ellipticity angle χ_2 ,

$$\psi_2 = 30 \text{ deg.} \quad (22.1)$$

$$\chi_2 = 0 \text{ deg.} \quad (22.2)$$

[0072] The angle that R_2 forms relative to the y_3 -axis is shown in Figure 15. This angle is,

$$\psi_{23} = \psi_2 - 60 = -30 \text{ deg.} \quad (22.3)$$

$$\psi = 90 - \psi_{23} \quad (22.4)$$

$$\tan\alpha_{23} = \{(1 - \cos 2\chi_2 \cos 2\psi)/(1 + \cos 2\chi_2 \cos 2\psi)\}^{1/2} \quad (22.5)$$

$$= -1.732$$

$$\cos\delta_{23} = \{(1 - \tan^2\alpha_{23})/\tan\alpha_{23}\}\tan 2\psi_{23}/2 \quad (22.6)$$

$$\delta_{23} = 180 \text{ deg.}$$

Reflection 3

[0073] For reflection 3,

$$\Delta = 0 \text{ deg.} \quad (22.7)$$

$$\tan\alpha = \tan\alpha_{23} = -1.732. \quad (22.8)$$

$$\cos(\Delta + \delta_{23}) = \{(1 - \tan^2\alpha)/\tan\alpha\}\tan 2\psi_3/2 \quad (23)$$

$$\sin 2\chi_3 = \{2\tan\alpha/(1 + \tan^2\alpha)\}\sin(\Delta + \delta_{23}) \quad (24)$$

Solving equation 23 for the azimuth angle ψ_3 and equation 24 for the ellipticity angle χ_3 ,

$$\psi_3 = -30 \text{ deg.} \quad (24.1)$$

$$\chi_3 = 0 \text{ deg.} \quad (24.2)$$

[0074] As shown in Fig. 16A, the azimuth angle relative to laboratory frame is given by,

$$\psi_{lab} = 30 + \psi_3 \quad (24.3)$$

$$= 0 \text{ deg.}$$

[0075] This is the exact result that was obtained in Case I, where all sides were coated with zero degree phase difference coating (in Table 2). Therefore, Cases I, II and III all produce the same result for linearly polarized input states.

Case IV: Additional designs of phase-compensating film stack

[0076] In one embodiment, the thin film stack on the cube corner induces 0 degree phase difference on reflection (i.e., $\Delta = 180$ degrees). The coating design is shown in Table 4. Fig. 16B illustrates the phase change with the coating of Table 4.

Layer#	Material	Index (at 633 nm)	QWOT (nm)
1	MgF2	1.38	715
2	TiO2	2.45	1903

Table 4 - 0 degree phase difference

Here layer 1 is the first layer on the BK-7 surface.

[0077] In another embodiment, the thin film stack on the cube corner induces 0 degree phase difference on reflection. The coating design is shown in Table 5. Please note that the lower TiO2 index is a result of a deposition process without ion-assistance. Fig. 16C illustrates the phase change with the coating of Table 5.

Layer#	Material	Index (at 633 nm)	QWOT (nm)
1	TiO2	2.30	262.5
2	SiO2	1.45	346.5
3	TiO2	2.30	1018.5
4	SiO2	1.45	462
5	TiO2	2.30	850.5

Table 5 - 0 degree phase difference

Here layer 1 is the first layer on the BK-7 surface.

[0078] For linearly polarized input states, the coating designs of Tables 4 and 5 would produce the desired result if all three faces or only the first and the third faces are coated. For circularly polarized input states, the coating designs of Tables 4 and 5 would produce the desired result if all three faces are coated.

Application to Interferometry Systems

Plane mirror interferometer

[0079] Fig. 17A illustrates a plane mirror interferometer system 100 in one embodiment of the invention. System 100 is similar to system 20 of Fig. 2 except that the conventional cube corner 10 has been replaced with a CC retroreflector 102 having phase-compensated interference film stacks 104. Depending on the embodiment, phase-compensated CC retroreflector 102 can be any of the CC retroreflectors with a phase-compensating film stack deposited on all three reflecting faces or only the first and the third reflecting faces as described above. As CC retroreflector 102 only handles linearly polarized input states, any film stack that induces $n\pi$ degree phase difference upon reflection can be used (e.g., 0 degree phase difference and $\Delta = 180$ degrees, and 180 degrees phase difference and $\Delta = 0$ degree) when all three faces are coated. A film stack that induces $2n\pi$ degree phase difference upon reflection can be used when all three faces or only the first and the third reflecting faces are coated. Whenever $n\pi$ and $2n\pi$ are mentioned, n is an integer including 0.

[0080] Although not shown, a 45 degree mirror can be placed in the reference path so reference mirror 30 is parallel to measurement mirror 26. The 45 degree mirror would be interposed between polarizing beam-splitter 24 and quarter-wave plate 32. The reference mirror 30 can be mounted to a moving component to provide a differential measurement relative to measurement mirror 26.

[0081] Fig. 17B illustrates an interferometer system 120 in one embodiment of the invention. System 120 is similar to system 100 of Fig. 17A except that measurement mirror 26 has been replaced with a phase-compensated CC retroreflector 122. The measurement path has changed as follows.

[0082] In the measurement path; polarizing beam-splitter 24 transmits frequency component f_A to phase-compensated cube corner 122 mounted to a moving stage. Since frequency component f_A passes through a quarter-wave plate 28, the returning polarization is rotated 90 degrees and the newly S-polarized frequency component f_A is reflected by polarizing beam-splitter 24 to phase-compensated CC retroreflector 102. Note that after passing through quarter-wave plate 28, component f_A is circularly polarized when it is incident on CC retroreflector 122.

[0083] CC retroreflector 102 directs frequency component f_A again to polarizing beam-splitter 24, which again reflects frequency component f_A to CC retroreflector 122. Again, since frequency component f_A passes through quarter-wave plate 28, the returning polarization is rotated 90 degrees and the newly P-polarized frequency component f_A is transmitted through polarizing beam-splitter 24 onto a receiver 34. Note again that after passing through quarter-wave plate 28, component f_A is circularly polarized when it is incident on CC retroreflector 122.

[0084] Depending on the embodiment, CC retroreflector 122 can be any of the CC retroreflectors with a phase-compensating film stack that induces $2n\pi$ degree phase difference upon reflection deposited on all three reflecting faces as described above (e.g., 0 degree phase difference and $\Delta = 180$ degrees). This is because CC retroreflector 142 must return a right circularly polarized light as a left circularly polarized light (or vice versa) so that after passing through quarter-wave plate 28, polarizing beam-splitter 24 would send the light onto the correct path. The handedness of the polarization is defined with the beam coming towards the observer.

[0085] Fig. 17C illustrates an interferometer system 140 in one embodiment of the invention. System 140 is similar to system 120 of Fig. 17B except that CC retroreflector 122 has been replaced with phase-compensated CC retroreflectors 142 and 144. The measurement path has changed as follows.

[0086] Polarizing beam-splitter 24 transmits frequency component f_A to phase-compensated CC retroreflector 142 mounted to a moving stage. Since frequency component f_A passes through a quarter-wave plate 28, the returning polarization is rotated 90 degrees and the newly S-polarized frequency component f_A is reflected by polarizing beam-splitter 24 to phase-compensated CC retroreflector 102. Note that after passing through quarter-wave plate

28, component f_A is circularly polarized when it is incident on CC retroreflector 142.

[0087] CC retroreflector 102 directs frequency component f_A again to polarizing beam-splitter 24, which reflects frequency component f_A to phase-compensated CC retroreflector 144. Again, since frequency component f_A passes through quarter-wave plate 28, the returning polarization is rotated 90 degrees and the newly P-polarized frequency component f_A is transmitted through polarizing beam-splitter 24 onto a receiver 34. Note that after passing through quarter-wave plate 28, component f_A is circularly polarized when it is incident on CC retroreflector 144.

[0088] Depending on the embodiment, CC retroreflectors 142 and 144 can be any of the CC retroreflectors with a phase-compensating film stack that causes $2n\pi$ degree phase difference upon reflection deposited on all three reflecting faces as described above. This is because CC retroreflectors 142 and 144 must return a right circularly polarized light as a left circularly polarized light (or vice versa) so that after passing through quarter-wave plate 28, polarizing beam-splitter 24 would send the light onto the correct path. The handedness of the polarization is defined with the beam coming towards the observer.

Differential Interferometer System

[0089] Figs. 18A, 18B, and 18C illustrate a differential interferometer system 200 in one embodiment of invention. A laser head 202 generates a coherent, collimated light beam consisting of two orthogonally polarized frequency components. One frequency component f_A (e.g., a measurement beam having a P-polarization) enters the interferometer's measurement path while the other frequency component f_B (e.g., a reference beam having an S-polarization) enters the interferometer's reference path.

[0090] In the measurement path, a polarizing beam-splitter 204 transmits frequency component f_A to a measurement plane mirror 206 mounted to a moving stage. Since frequency component f_A passes through a quarter-wave plate 208, the returning polarization is rotated 90 degrees and the newly S-polarized frequency component f_A is reflected by polarizing beam-splitter 204 to a phase-compensated CC retroreflector 210. CC retroreflector 210 directs frequency component f_A again to polarizing beam-splitter 204, which again reflects frequency component f_A to measurement plane mirror 206. Again, since frequency component f_A passes through quarter-wave plate 208, the returning polarization is rotated 90 degrees and the newly P-polarized frequency component f_A is transmitted through polarizing

beam-splitter 204 on a mirror 212.

[0091] Mirror 212 directs frequency component f_A to a phase-compensated CC retroreflector 214, which returns frequency component f_A back to mirror 212. Since frequency component f_A passes through quarter-wave plate 216, the returning polarization is rotated 90 degrees. Mirror 212 then returns the newly S-polarized frequency component f_A to polarizing beam-splitter 204, which reflects frequency component f_A to a phase-compensated CC retroreflector 218. CC retroreflector 218 returns frequency component f_A to polarizing beam-splitter 204, which reflects frequency component f_A to a receiver 220.

[0092] In the reference path, polarizing beam-splitter 204 reflects frequency component f_B to phase-compensated CC retroreflector 218. Phase-compensated CC retroreflector 218 returns frequency component f_B to polarizing beam-splitter 204, which reflects frequency component f_B to mirror 212. Mirror 212 directs frequency component f_B to phase-compensated CC retroreflector 214, which returns frequency component f_B back to mirror 212. Since frequency component f_B passes through quarter-wave plate 216, the returning polarization is rotated 90 degrees.

[0093] Mirror 212 directs the newly P-polarized frequency component f_B to polarizing beam-splitter 204, which transmits frequency component f_B to a reference mirror 222 that may move. Since frequency component f_B passes through a quarter-wave plate 208, the returning polarization is rotated 90 degrees and the newly S-polarized frequency component f_B is reflected by polarizing beam-splitter 204 to phase-compensated CC retroreflector 210. CC retroreflector 210 returns frequency component f_B to polarizing beam-splitter 204, which again reflects frequency component f_B to reference mirror 222. Again, since frequency component f_B passes through quarter-wave plate 208, the returning polarization is rotated 90 degrees and the P-polarized frequency component f_A is transmitted through polarizing beam-splitter 204 onto receiver 220.

[0094] Depending on the embodiment, phase-compensated CC retroreflector 214 can be any of the CC retroreflectors with a phase-compensating film stack that induces $2n\pi$ degree difference upon reflection deposited on all three reflecting faces as described above. This is because it must return a right circularly polarized light as a left circularly polarized light (or vice versa) so that after passing through quarter-wave plate 216, polarizing beam-splitter 204 would send the light onto the correct path. The handedness of the polarization is defined with

the beam coming towards the observer.

[0095] Depending on the embodiment, phase compensated CC retroreflectors 210 and 218 can be any of the CC retroreflectors with a phase-compensating film stack deposited on all three reflecting faces. The film stack is deposited on all three reflecting faces because components f_A and f_B may strike the reflecting faces of the retroreflectors in different orders. As CC retroreflectors 210 and 218 handle only linearly polarized input states, any film stack that induces $n\pi$ degree difference upon reflection can be used.

Single Beam Interferometer System

[0096] Fig. 19 illustrates a single beam interferometer system 300 in one embodiment of the invention. A laser head 302 generates a coherent, collimated light beam consisting of two orthogonally polarized frequency components. One frequency component f_A (e.g., a measurement beam having a P-polarization) enters the interferometer's measurement path while the other frequency component f_B (e.g., a reference beam having an S-polarization) enters the interferometer's reference path.

[0097] In the measurement path, a polarizing beam-splitter 304 transmits frequency component f_A to a phase-compensated CC retroreflector 306 mounted to a moving stage. Since frequency component f_A passes through a quarter-wave plate 308, the returning polarization is rotated 90 degrees and the newly S-polarized frequency component f_A is reflected by polarizing beam-splitter 304 to a receiver 310.

[0098] In the reference path, polarizing beam-splitter 304 reflects frequency component f_B to a phase-compensated CC retroreflector 312. Since frequency component f_B passes through a quarter-wave plate 314, the returning polarization is rotated 90 degrees and the newly P-polarized frequency component f_B is transmitted through polarizing beam-splitter 304 to receiver 310.

[0099] Depending on the embodiment, phase-compensated CC retroreflectors 306 and 312 can be any of the CC retroreflectors with a phase-compensating film stack that induces $2n\pi$ phase difference upon reflection deposited on all three reflecting faces as described above. This is because CC retroreflectors 306 and 312 must return a right circularly polarized light as a left circularly polarized light (or vice versa) so that after passing through corresponding quarter-wave plates 308 and 314, polarizing beam-splitter 304 would send the light onto the

correct path. The handedness of the polarization is defined with the beam coming towards the observer.

High-Resolution Interferometer System

[0100] Figs. 20A, 20B, and 20C illustrate a high-resolution interferometer system 400 in one embodiment of invention. A laser head 402 generates a coherent, collimated light beam consisting of two orthogonally polarized frequency components. One frequency component f_A (e.g., a measurement beam having a P-polarization) enters the interferometer's measurement path while the other frequency component f_B (e.g., a reference beam having an S-polarization) enters the interferometer's reference path.

[0101] In the measurement path, a polarizing beam-splitter 404 transmits frequency component f_A to a measurement plane mirror 406 mounted to a moving stage. Since frequency component f_A passes through a quarter-wave plate 408, the returning polarization is rotated 90 degrees and the newly S-polarized frequency component f_A is reflected by polarizing beam-splitter 404 to a phase-compensated CC retroreflector 410. CC retroreflector 410 directs frequency component f_A again to polarizing beam-splitter 404, which again reflects frequency component f_A to measurement plane mirror 406. Again, since frequency component f_A passes through quarter-wave plate 408, the returning polarization is rotated 90 degrees and the newly P-polarized frequency component f_A is transmitted through polarizing beam-splitter 404 on a mirror 412.

[0102] Mirror 412 directs frequency component f_A to a phase-compensated CC retroreflector 414, which returns frequency component f_A back to mirror 412. Mirror 412 then returns frequency component f_A to polarizing beam-splitter 204, which transmits frequency component f_A to plane mirror 406. Again, since frequency component f_A passes through quarter-wave plate 408, the returning polarization is rotated 90 degrees and the newly S-polarized frequency component f_A is reflected by polarizing beam-splitter 404 to phase-compensated CC retroreflector 410. CC retroreflector 410 directs frequency component f_A again to polarizing beam-splitter 404, which again reflects frequency component f_A to measurement plane mirror 406. Again, since frequency component f_A passes through quarter-wave plate 408, the returning polarization is rotated 90 degrees and the newly P-polarized frequency component f_A is transmitted through polarizing beam-splitter 404 to a receiver 416.

[0103] In the reference path, polarizing beam-splitter 404 reflects frequency component f_B to a mirror 418. Since frequency component f_A passes through quarter-wave plate 420, the returning polarization is rotated 90 degrees and the newly P-polarized frequency component f_A is transmitted through polarizing beam-splitter 404 to phase-compensated CC retroreflector 410. CC retroreflector 410 returns frequency component f_A to polarizing beam-splitter 404, which transmits frequency component f_A to mirror 418. Again, since frequency component f_A passes through quarter-wave plate 420, the returning polarization is rotated 90 degrees and the newly S-polarized frequency component f_A is reflected by polarizing beam-splitter 404 to mirror 412.

[0104] Mirror 412 directs frequency component f_A to phase-compensated CC retroreflector 414, which returns frequency component f_A back to mirror 412. Mirror 412 then returns frequency component f_A to polarizing beam-splitter 204, which reflects frequency component f_A to mirror 420. Since frequency component f_A passes through quarter-wave plate 420, the returning polarization is rotated 90 degrees and the newly P-polarized frequency component f_A is transmitted by polarizing beam-splitter 404 to phase-compensated CC retroreflector 410. CC retroreflector 410 returns frequency component f_A to polarizing beam-splitter 404, which transmits frequency component f_A to mirror 418. Again, since frequency component f_A passes through quarter-wave plate 420, the returning polarization is rotated 90 degrees and the newly S-polarized frequency component f_A is reflected by polarizing beam-splitter 404 to receiver 416.

[0105] Depending on the embodiment, phase-compensated CC retroreflector 414 can be any of the CC retroreflectors with a phase-compensating film stack deposited on all three reflecting faces or only the first and the third faces as described above. The phase-compensating film stack that induces $n\pi$ degree phase difference upon reflection can be used when all three faces are coated. A film stack that induces $2n\pi$ degree phase difference upon reflection can be used when all three faces or only the first and the third reflecting faces are coated.

[0106] Depending on the embodiment, phase-compensated CC retroreflector 410 can be any of the CC retroreflectors with a phase-compensating film stack deposited on all three reflecting faces. The film stack is deposited on all three faces because components f_A and f_B each makes two passes through the cube corner and may strike the faces in different orders. As CC retroreflector 410 only handle linearly polarized input states, any phase-compensating

interference film that induces $n\pi$ phase difference upon reflection can be used.

Linear Interferometer System

[0107] Fig. 21 illustrates a linear interferometer system 500 in one embodiment of the invention. A laser head 502 generates a coherent, collimated light beam consisting of two orthogonally polarized frequency components. One frequency component f_A (e.g., a measurement beam having a P-polarization) enters the interferometer's measurement path while the other frequency component f_B (e.g., a reference beam having an S-polarization) enters the interferometer's reference path.

[0108] In the measurement path, a polarizing beam-splitter 504 transmits frequency component f_A to a phase-compensated CC retroreflector 506 mounted to a moving stage. Phase-compensated retroreflector 506 returns frequency component f_A to polarizing beam splitter 504, which transmits the frequency component f_A to a receiver 508.

[0109] In the reference path, polarizing beam-splitter 504 reflects frequency component f_B to a phase-compensated CC retroreflector 510. Phase-compensated retroreflector 510 returns frequency component f_B to polarizing beam splitter 504, which reflects the frequency component f_A to receiver 508.

[0110] Depending on the embodiment, phase-compensated CC retroreflectors 506 and 510 can be any of the CC retroreflectors with a phase-compensating film stack deposited on all three reflecting faces or only the first and the third reflecting faces as described above. The phase-compensating film stack that induces $n\pi$ degree phase difference upon reflection can be used when all three faces are coated. A film stack that induces $2n\pi$ degree phase difference upon reflection can be used when all three faces or only the first and the third reflecting faces are coated.

Summary

[0111] With the aid of a numerical model, polarization effects from a TIR cube corner were analyzed. This model predicts significant changes to the azimuth and ellipticity angles of incident polarization from an uncoated cube corner. As a consequence, multilayer interference coatings were designed to produce zero and 180 degrees phase difference between S and P states for TIR reflection. The results indicate that with all or only first and

third faces coated, the azimuth and ellipticity angles of the initial polarization were preserved.

[0112] Various other adaptations and combinations of features of the embodiments disclosed are within the scope of the invention. Although the various interferometry systems described above use vertically and horizontally polarized light, the interferometry systems can also use left and right circularly polarized light. Although the various interferometry systems described above show measurement along a single axis, it is understood that these systems could be adopted for multi-axes measurement by replicating the setups or integrating the single-axis measurement components into a multi-axes module. Although specific coating designs for BK-7 cube corners are disclosed, one skilled in the art can modify the coating designs for cube corners made of other materials such as fused silica, SF-10, and etc. Furthermore, one skilled in the art can modify the coating designs to suit other operating wavelengths. Numerous embodiments are encompassed by the following claims.

Appendix I

[0113] For a monochromatic plane wave E propagating in the z direction with coordinates E_x and E_y ,

$$E_x = a_1 \cos(\tau + \delta_1) \quad \text{I.1.a}$$

$$E_y = a_2 \cos(\tau + \delta_2) \quad \text{I.1.b}$$

$$E_z = 0, \quad \text{I.1.c}$$

where a_1 is the amplitude along the x -axis, δ_1 is the phase along the x -axis (please note this is different from the relative phase difference δ_1 after reflection from the first face described above), a_2 is the amplitude along the y -axis, δ_2 is the phase along the y -axis (please note this is different from the relative phase difference δ_2 after reflection from the second face described above), $\tau = \omega t - kz$, ω is angular frequency, t is time, k is wave number ($2\pi/\lambda$), and z is propagation direction.

[0114] Rewriting (I.1.a) and (I.1.b),

$$E_x/a_1 = \cos(\tau + \delta_1) = \cos\tau\cos\delta_1 - \sin\tau\sin\delta_1 \quad \text{I.1.d}$$

$$E_y/a_2 = \cos(\tau + \delta_2) = \cos\tau\cos\delta_2 - \sin\tau\sin\delta_2. \quad \text{I.1.e}$$

[0115] To eliminate τ dependence, we multiply each component by $\sin\delta_1$, $\sin\delta_2$, $\cos\delta_1$ and $\cos\delta_2$:

$$(E_x/a_1)\sin\delta_2 = \sin\delta_2(\cos\tau\cos\delta_1 - \sin\tau\sin\delta_1) \quad \text{I.2.a}$$

$$(E_y/a_2)\sin\delta_1 = \sin\delta_1(\cos\tau\cos\delta_2 - \sin\tau\sin\delta_2) \quad \text{I.2.b}$$

$$(E_x/a_1)\cos\delta_2 = \cos\delta_2(\cos\tau\cos\delta_1 - \sin\tau\sin\delta_1) \quad \text{I.2.c}$$

$$(E_y/a_2)\cos\delta_1 = \cos\delta_1(\cos\tau\cos\delta_2 - \sin\tau\sin\delta_2) \quad \text{I.2.d}$$

[0116] Subtracting (I.2.b) from (I.2.a),

$$(E_x/a_1)\sin\delta_2 - (E_y/a_2)\sin\delta_1 = \cos\tau\sin(\delta_2 - \delta_1). \quad \text{I.3.a}$$

[0117] Subtracting (I.2.d) from (I.2.c),

$$(E_x/a_1)\cos\delta_2 - (E_y/a_2)\cos\delta_1 = \sin\tau\sin(\delta_2 - \delta_1). \quad \text{I.3.b}$$

[0118] Squaring and adding (I.3.a) and (I.3.b) yields,

$$(E_x/a_1)^2 + (E_y/a_2)^2 - 2(E_x/a_1)(E_y/a_2)(\sin\delta_2\sin\delta_1 + \cos\delta_1\cos\delta_2) = \sin^2(\delta_2 - \delta_1)$$

and simplifying,

$$(E_x/a_1)^2 + (E_y/a_2)^2 - 2(E_x/a_1)(E_y/a_2)\cos(\delta_2 - \delta_1) = \sin^2(\delta_2 - \delta_1). \quad \text{I.4}$$

Appendix II

[0119] In the rotated coordinate system (ξ, η) , the components of the electric vector are related by,

$$E_\xi = E_x\cos\psi + E_y\sin\psi \quad \text{II.1.a}$$

$$E_\eta = -E_x\sin\psi + E_y\cos\psi, \quad \text{II.1.b}$$

where ψ is the angle between (ξ, η) and (x, y) coordinate axes. In the (ξ, η) coordinate system, the equation for the electric vector is given by,

$$E_\xi = a*\cos(\tau + \delta_\xi) \quad \text{II.2.a}$$

$$E_\eta = b*\cos(\tau + \delta_\eta), \quad \text{II.2.b}$$

where a is the amplitude along the ξ -axis, b is the amplitude along the η -axis, and $\tau = \omega t - kz$. Then using the results of equation II.2.a and II.2.b, the expressions for the electric vector

reduces to,

$$E_z = a \cos(\tau + \delta_z) \quad \text{II.2.c}$$

$$E_y = \pm b \sin(\tau + \delta_y). \quad \text{II.2.d}$$

[0120] Now a relationship between the amplitudes a , b of the rotated frame and a_1 , a_2 of the x , y frame is established. First equations I.1.a, I.1.b, II.2.c and (II.2.d) are expanded as follows:

$$E_x = a_1 \cos(\tau + \delta_1) = a_1 (\cos\tau \cos\delta_1 - \sin\tau \sin\delta_1) \quad \text{II.3.a}$$

$$E_y = a_2 \cos(\tau + \delta_2) = a_2 (\cos\tau \cos\delta_2 - \sin\tau \sin\delta_2) \quad \text{II.3.b}$$

$$E_z = a \cos(\tau + \delta_z) = a (\cos\tau \cos\delta_z - \sin\tau \sin\delta_z) \quad \text{II.3.c}$$

$$E_y = \pm b \sin(\tau + \delta_y) = \pm b (\sin\tau \cos\delta_y + \cos\tau \sin\delta_y). \quad \text{II.3.d}$$

[0121] Combining equation II.1.a and II.1.b with equations II.3.a and II.3.b,

$$E_z = a_1 (\cos\tau \cos\delta_1 - \sin\tau \sin\delta_1) \cos\psi + a_2 (\cos\tau \cos\delta_2 - \sin\tau \sin\delta_2) \sin\psi \quad \text{II.4.a}$$

$$E_y = -a_1 (\cos\tau \cos\delta_1 - \sin\tau \sin\delta_1) \sin\psi + a_2 (\cos\tau \cos\delta_2 - \sin\tau \sin\delta_2) \cos\psi \quad \text{II.4.b}$$

then equating the corresponding terms of equations II.3.c, II.3.d and equations II.4.a, II.4.b:

For E_z equation,

cost terms:

$$a \cos\delta_z = a_1 \cos\delta_1 \cos\psi + a_2 \cos\delta_2 \sin\psi \quad \text{II.5.a}$$

sint terms:

$$a \sin\delta_z = a_1 \sin\delta_1 \cos\psi + a_2 \sin\delta_2 \sin\psi \quad \text{II.5.b}$$

For E_y equations,

cost terms:

$$\pm b \sin\delta_y = -a_1 \cos\delta_1 \sin\psi + a_2 \cos\delta_2 \cos\psi \quad \text{II.6.a}$$

sint terms:

$$\pm b \cos\delta_y = a_1 \sin\delta_1 \sin\psi - a_2 \sin\delta_2 \cos\psi \quad \text{II.6.b}$$

[0122] Now squaring and adding equations II.5.a and II.5.b,

$$a^2 = a_1^2 \cos^2\psi + a_2^2 \sin^2\psi + 2a_1 a_2 \cos\psi \sin\psi \cos(\delta_2 - \delta_1), \quad \text{II.7.a}$$

and performing the same operations with equations II.6.a and II.6.b,

$$b^2 = a_1^2 \sin^2 \psi + a_2^2 \cos^2 \psi - 2a_1 a_2 \sin \psi \cos \psi \cos(\delta_2 - \delta_1). \quad \text{II.7.b}$$

[0123] Adding (II.7.a) and (II.7.b),

$$a^2 + b^2 = a_1^2 + a_2^2. \quad \text{II.8}$$

[0124] Next equations II.5.a and II.6.b are multiplied together and equations II.5.b and II.6.a are also multiplied together as follows:

$$+/-ab \cdot \cos^2 \delta_i = -a_1 a_2 \sin \delta_2 \cos \delta_1 \cos^2 \psi + a_1 a_2 \sin \delta_1 \cos \delta_2 \sin^2 \psi \quad \text{II.9.a}$$

$$+/-ab \cdot \sin^2 \delta_i = -a_1 a_2 \sin \delta_2 \cos \delta_1 \sin^2 \psi + a_1 a_2 \sin \delta_1 \cos \delta_2 \cos^2 \psi \quad \text{II.9.b}$$

then adding (II.9.a) and (II.9.b),

$$+/-ab = a_1 a_2 \sin(\delta_1 - \delta_2)$$

or alternatively,

$$-/+ab = a_1 a_2 \sin(\delta_2 - \delta_1). \quad \text{II.10}$$

[0125] Furthermore, dividing equation II.6.b by equation II.5.a, and dividing equation II.6.a by equation II.5.b yields:

$$+/-b/a = (a_1 \sin \delta_1 \sin \psi - a_2 \sin \delta_2 \cos \psi) / (a_1 \cos \delta_1 \cos \psi + a_2 \cos \delta_2 \sin \psi) \quad \text{II.11.a}$$

$$+/-b/a = (-a_1 \cos \delta_1 \sin \psi + a_2 \cos \delta_2 \cos \psi) / (a_1 \sin \delta_1 \cos \psi + a_2 \sin \delta_2 \sin \psi), \quad \text{II.11.b}$$

and equating the right hand side (RHS) of equations II.11.a and II.11.b,

$$(a_1^2 - a_2^2) \sin 2\psi = 2a_1 a_2 \cos(\delta_2 - \delta_1) \cos 2\psi$$

or equivalently,

$$(a_1^2 - a_2^2) \tan 2\psi = 2a_1 a_2 \cos \delta \quad \text{II.12}$$

where $\delta = \delta_2 - \delta_1$.

[0126] The last relation is derived by multiplying equation II.10 by 2 and then dividing by equation II.8,

$$-/+2ab/(a^2 + b^2) = 2a_1a_2\sin\delta/(a_1^2 + a_2^2) \quad \text{II.13}$$

Appendix III

[0127] To derive the expressions for the Stokes parameters, let us recall equations II.8, II.12, and II.13,

$$a^2 + b^2 = a_1^2 + a_2^2. \quad \text{(II.8)}$$

$$(a_1^2 - a_2^2)\tan 2\psi = 2a_1a_2\cos\delta \quad \text{(II.12)}$$

$$-/+2ab/(a^2 + b^2) = 2a_1a_2\sin\delta/(a_1^2 + a_2^2) \quad \text{(II.13)}$$

and define

$$\tan\alpha = a_2/a_1 \quad \text{(III.0.a)}$$

$$\tan\chi = +/-b/a. \quad \text{(III.0.b)}$$

[0128] From equation II.12, we make the following definitions:

Right Hand Side Definitions (RHS):

$$S_1 = a_1^2 - a_2^2 \quad \text{III.1}$$

$$S_2 = 2a_1a_2\cos\delta \quad \text{III.2}$$

Left Hand Side Definition (LHS):

$$S_2 = (a_1^2 - a_2^2)\tan 2\psi \quad \text{III.3}$$

[0129] Similarly from equation II.13, we have the following definitions:

Right Hand Side Definition (RHS):

$$S_3 = 2a_1a_2\sin\delta \quad \text{III.4}$$

Left Hand Side Definition (LHS):

$$S_3 = -/+2ab(a_1^2 + a_2^2)/(a^2 + b^2) \quad \text{III.5}$$

[0130] Next defined is a dependent parameter S_0 . This is defined by combining the squares of the RHS definitions of S_1 , S_2 and S_3 ,

Right Hand Side Definition (RHS):

$$\begin{aligned} S_0^2 &= S_1^2 + S_2^2 + S_3^2 \\ &= (a_1^2 - a_2^2)^2 + (2a_1a_2\cos\delta)^2 + (2a_1a_2\sin\delta)^2 \\ &= (a_1^2 + a_2^2)^2 \\ S_0 &= a_1^2 + a_2^2 \end{aligned} \quad \text{III.6}$$

Left Hand Side Definition (LHS):

$$S_0^2 = S_1^2 + S_2^2 + S_3^2 \quad \text{III.7}$$

[0131] Taking next the LHS definition of S_3 ,

$$S_3 = -/+2ab(a_1^2 + a_2^2)/(a^2 + b^2),$$

or alternatively with the substitution of equation III.6, S_3 becomes,

$$S_3 = -/+2abS_0/(a^2 + b^2).$$

[0132] Further, note that $\tan\chi = -/+b/a$; and thus S_3 may be expressed as,

$$\begin{aligned} S_3 &= 2S_0\tan\chi/(1 + \tan^2\chi) = S_0\sin 2\chi \\ S_3 &= S_0\sin 2\chi. \end{aligned} \quad \text{III.8}$$

[0133] Now to express S_1 and S_2 in terms of the parameters S_0 , ψ and χ , we take the LHS definition of equation III.3 and rewrite it as,

$$S_2 = S_1\tan 2\psi. \quad \text{III.9}$$

[0134] Then substituting equations III.8 and III.9 into equation III.7,

$$S_0^2 = S_1^2 + (S_1 \tan 2\psi)^2 + (S_0 \sin 2\chi)^2$$

and solving for S_1 ,

$$S_1 = S_0 \cos 2\psi \cos 2\chi. \quad \text{III.10}$$

[0135] Now substitute equation III.10 into equation III.9 to arrive at a new form of S_2 ,

$$S_2 = S_0 \sin 2\psi \cos 2\chi. \quad \text{III.11}$$

[0136] Summarizing the results,

$$\begin{aligned} S_0^2 &= S_1^2 + S_2^2 + S_3^2 \\ S_1 &= S_0 \cos 2\psi \cos 2\chi \\ S_2 &= S_0 \sin 2\psi \cos 2\chi \\ S_3 &= S_0 \sin 2\chi. \end{aligned}$$

[0137] Expressing the Stokes parameters with respect to RHS definitions,

$$\begin{aligned} S_0 &= a_1^2 + a_2^2 \\ S_1 &= a_1^2 - a_2^2 \\ S_2 &= 2a_1 a_2 \cos \delta \\ S_3 &= 2a_1 a_2 \sin \delta. \end{aligned}$$

[0138] These RHS Stokes parameters are then normalize by dividing by S_0 .

$$\begin{aligned} s_0 &= 1 \\ s_1 &= (a_1^2 - a_2^2)/(a_1^2 + a_2^2) \\ s_2 &= 2a_1 a_2 \cos \delta / (a_1^2 + a_2^2) \\ s_3 &= 2a_1 a_2 \sin \delta / (a_1^2 + a_2^2). \end{aligned}$$

[0139] Then expressing in terms of $\tan \alpha = a_2/a_1$,

$$s_0 = 1 \quad (III.12)$$

$$s_1 = (1 - \tan^2 \alpha) / (1 + \tan^2 \alpha) \quad (III.13)$$

$$s_2 = 2 \tan \alpha \cos \delta / (1 + \tan^2 \alpha) \quad (III.14)$$

$$s_3 = 2 \tan \alpha \sin \delta / (1 + \tan^2 \alpha) \quad (III.15)$$